LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2023**

PMT 3602 – DIFFERENTIAL GEOMETRY

Date: 08-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Answer ALL the questions

1. (a) Find the centre and radius of the osculating circle. (5)

(OR)

(b) Find the length of the circular helix $\vec{r} = acosu\,\vec{\imath} + asinu\,\vec{\jmath} + bu\,\vec{k}$, $-\infty < u < \infty$ varies from the point $(a, 0,0)$ to $(a, 0,2\pi b)$. Also obtain the equation in terms of parameter s. (5)

(c) Derive the equation of the osculating plane at a point on the intersection of two surfaces $f(x, y, z) = 0$ and $g(x, y, z) = 0$. (15)

(OR)

(d) State and prove Serret-Frenet formulae. (15)

2. (a) Find the inflexinol tangent at (x_1, y_1, z_1) on the surface $y^2z = 4ax$. (5)

(OR)

(b) Derive the equation of involute of a given curve. (5)

(c) State and prove Fundamental theorem of space curves. (15)

(OR)

(d) Show that the intrinsic equation of the curve $x = ae^u cos\,u$, $y = ae^u sin\,u$, $z = be^u$ are $k = \frac{a\sqrt{2}}{s\sqrt{2a^2+b^2}}$ and $\tau = \frac{b}{s\sqrt{2a^2+b^2}}$. (15)

3. (a) Write a brief note on transformation of parameters. (5)

(OR)

(b) Define (i) Artificial singularity (ii) Envelope (iii) Developable surface. (5)

(c) Explain the first fundamental form of a surface and give its geometrical interpretation. (15)

(OR)

(d) (i) Derive the equation of polar developable associated with a surface.

(ii) Show that the pole in the plane is artificial singularity. (10+5)

4. (a) State and prove Meusiner's theorem. (5)

(OR)

(b) Define (i) umbilic point (ii) total curvature (iii) line of curvature. (5)

(c) (i) Explain briefly the different points on a surface.

(ii) State and prove Euler's theorem. (5+10)

(OR)

(d) Derive the equation satisfying principal curvature and principal direction at a point on a surface.

(15)

5. (a) Derive Weingarten equations. (5)

(OR)

(b) If k_1 and k_2 are the principal curvatures and the lines of curvature are parametric curves then prove that the codazzi equations are $\frac{\partial k_1}{\partial v} = \frac{1}{2}\frac{E_v}{E}(k_2 - k_1)$ and $\frac{\partial k_2}{\partial u} = \frac{1}{2}\frac{G_u}{G}(k_1 - k_2)$.

(5)

(c) Derive Gauss equations of surface theory. (15)

(OR)

(d) State the Fundamental theorem of Surface Theory and demonstrate it in the case of unit sphere.

(15)

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